Name——————————, **ID** ————————

Final Exam MTH 221 , Summer 2011 at 12:30

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QUESTION 1. (Each = 4 points, Total = 76 points) Circle the correct letter.

(i) Given A is 2×2 and $A \ \overrightarrow{2R_1 + R_2 \to R_2} \ B$. Let E be an elementary matrix such that $EB = A$. Then $E =$

a) $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 0 \\ 0.5 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$ d) None is correct

(ii) Let $A \begin{bmatrix} 1 & 3 & 2 \\ -1 & b & b \\ -2 & -6 & b \end{bmatrix}$ such that $AX = \begin{bmatrix} 4 \\ 6 \\ -8 \end{bmatrix}$ has infinitely many solution. Then $b =$

a) -3 or -4 b) any real number except -3 and -4 c) -3 d) -4 e) None is correct

(iii) Let $A = \begin{bmatrix} 1 & 4 & a \\ -1 & 2 & b \\ 2 & 8 & c \end{bmatrix}$ and let B = First column of A + Second Column of A. Given $x_1 = -3, x_2 = 0.5, x_3 = 1$ is a solution to the system $AX = B$. Then

a) $det(A) \neq 0$ b) The system $AX = B$ has infinitely many solutions c) A is invertible d) (a) and (c) are correct

(iv) Let A, B, x_1, x_2, x_3 as above (as in PART iii). The third column of A is

a) $\begin{bmatrix} 5 \\ 1 \\ 10 \end{bmatrix}$ b) $\begin{bmatrix} 3 \\ 0 \\ 6 \end{bmatrix}$ c) $\begin{bmatrix} 6 \\ -3 \\ 12 \end{bmatrix}$ d) None is correct

(v) Let $A^{-1} = \begin{bmatrix} 4 & 2 & -2 \\ -4 & -3 & 0 \\ 2 & 1 & 0 \end{bmatrix}$ Then the solution to the system $AX = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ is

a) $x_1 = 6, x_2 = -7, x_3 = 3$ b) $x_1 = 4, x_2 = -4, x_3 = 2$ c) $x_1 = 10, x_2 = -11, x_3 = 5$ d) None is correct

(vi) Let A^{-1} as above. Then (3, 1)-entry of A is

a) 2.5 b)-0.5 -2.5 d) 0.5 e) None is correct

(vii) One of the following is true:

a) $\{A \in R_{2\times 2} \mid det(A) = 0\}$ is a subspace of $R_{2\times 2}$ b) $\{(a, b, c) \in R^3 \mid a, b, c \in R \ \ and \ \ a + b = 0 \ \ or \ \ a + c = 0\}$ is a subspace of R^3

c) $\{(a, b^2) \mid a, b \in R\}$ is a subspace of R^2 d) None is correct

(viii) Let A be 3×3 matrix and $C = A + I_3$ such that $x_1 = 2, x_2 = 1, x_3 = 0$ is a solution to the system $AX = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ and $x_1 = -2, x_2 = 1, x_3 = 0$ is a solution to the system $CX = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. The second column of A is

a) $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ b) Can not be determined c) $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ d) None is correct

(ix) Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & -1 & -1 & 4 \\ 2 & 2 & 2 & 2 \end{bmatrix}$ and let $T : R^4 \to R^3$ such that $T(a, b, c, d) = A \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$. Then Range(T) =

a)$Span\{(1, 1, 1, 1), (-1, -1, -1, 4)\}$ b) $Span\{(1, 5, 0), (1, 0, 0)\}$

c) $Span\{(1, -1, 2), (1, 4, 2)\}$ d) None is correct

(x) Let A and T as above. One of the following points lies in the Ker(T):

a) (1, 1, 0, 0) b) (0, 1, -1, 0) c) (1, -1, 0) d) (1, 0, 1, 0)

(xi) Let A and T as above. One of the following points lies in the range of T:

a)(5, -8, 0, 0) b) (0, 5, 0) c) (5,0, 0) d) (5, 0, 10)

(xii) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ -1 & -1 & 1 \end{bmatrix}$. The eigenvalues of A are :

a)1 b) 0, 1, 2 c) 0, 1 d) None is correct

(xiii) One of the following is a subspace of P_4

a) $\{a + bx + (2b + a)x^3 \mid a, b \in R\}$ b) $\{3 + ax + (a + 3)x^2 \mid a \in R\}$ c) $\{a^2 - bx \mid a, b \in R\}$

d) (a) and (c) are correct

(xiv) Let $T : P_4 \to R^3$ such that $T(p(x)) = (0, p'(1), p'(-1))$. Then $Ker(T) =$

a)$Span\{1, 10 - 3x + x^3\}$ b) R c) $span\{2x + 2x^2 - 6x^3\}$ d) $Span\{1 - 3x + x^3\}$

(xv) Let T as above. Then dim(Range(T)) =

a) 1 b) 3 c) 0 d)2

(xvi) Let $T : R^3 \to P_2$ such that $T(1, 0, 0) = x, T(0, 2, 0) = 2, T(0, 1, 1) = 1 + x$. Then T(3, 2, 4) =

a) 2x + 3 b) 7x + 2 c) 3x + 2 d) None is correct

(xvii) Let T as above. Then $Ker(T) =$

a) $Span\{(1, 1, 0)\}$ b) $Span\{(1, 0, 0), (0, 0, -1)\}$ $c)$Span {(1, 0, -1)} None is correct

(xviii) Let A be 3×3 such that $A \xrightarrow{2R_1} A_1 \xrightarrow{-3R_2 + R_3 \to R_3} \begin{bmatrix} 2 & 2 & 2 \\ -2 & 4 & -6 \\ -1 & -1 & 1 \end{bmatrix}$. Then $det(A) =$

a) 48 b) 24 c) 12 d)None is correct

(xix) Let A as above. The solution to the system $AX = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$ is

a) $x_1 = 0, x_2 = 1, x_3 = 1$ b) $x_1 = 0, x_2 = 2, x_3 = 0$ c)$x_1 = 4, x_2 = 1, x_3 = -1$ d) None is correct

QUESTION 2. (**7 points**)
Let $F = span\{(1,0,0,1), (0,0,-1,1), (0,1,0,1)\}$ Find an orthogonal basis for F

QUESTION 3. (**5 points**) Find two matrices A, B such that each is 2×2 matrix and $det(A) = det(B) = -6$ but $det(A+B) = -20$. If no such matrices exist, then explain

QUESTION 4. (**12 points**) Let $A = \begin{bmatrix} 2 & 1 & 1 & -6 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ If possible find a diagonal matrix D and invertible matrix Q such that $A = QDQ^{-1}$ (Do not calculate Q^{-1})

Faculty information

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